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OM theory and V-duality

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Abstract: We show that the (M5, M2, M2′, MW) bound state solution of eleven-dimensional supergravity recently constructed in hep-th/0009147 is related to the (M5, M2) bound state one by a finite Lorentz boost along a M5-brane direction perpendicular to the M2-brane. Given the (M5, M2) bound state as a defining system for OM theory and the above relation between this system and the (M5, M2, M2′, MW) bound state, we test the recently proposed V-duality conjecture in OM theory. Insisting to have a decoupled OM theory, we find that the allowed Lorentz boost has to be infinitesimally small, therefore resulting in a family of OM theories related by Galilean boosts. We argue that such related OM theories are equivalent to each other. In other words, V-duality holds for OM theory as well. Upon compactification on either an electric or a "magnetic" circle (plus T-dualities as well), the V-duality for OM theory gives the known one for either non-commutative open string theories or non-commutative Yang-Mills theories. This further implies that V-duality holds in general for the little m-theory without gravity.

Keywords: M-Theory, p-branes, String Duality.

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Contents

1. Introduction **1**

2. The (M5, M2, M2′, MW) bound state **3**

3. The galilean nature of OM theory: V-duality **5**

4. Compactifications of the deformed OM theory **8**

4.1 Compactification on an electric circle 9

4.2 Compactification on a magnetic circle 10

5. Conclusion **12**

1. Introduction

Recently there is a surge of interest in studying the effect of a constant background Neveu-Schwarz (NS) B-field on the decoupled theory of D-branes in superstring theory. It has been found that the worldvolume coordinates of Dp-branes can become non commutative along the directions of a non-vanishing B-field. When the non-zero B-field is space-like, one can define a decoupling limit for a non-commutative Yang-Mills theory (NCYM), i.e. a non-commutative field theory [1]. On the other hand, if the B-field is time-like, a so-called non-commutative open string (NCOS) theory can be achieved [2, 3].

The counterpart of NS B-field in M-theory is a three-form C-field. Here we have an open M2-brane ending on M5-branes which plays a similar role as an open string ending on Dp-branes does in superstring theory. So a natural question is to ask whether there exists a decoupled theory in M-theory in which the C-field plays a similar role as the NS B-field does to the decoupled theories of D-branes in string theory [1], [4]–[8]. Indeed it has been found that a near-critical electric field C_{012} defines a decoupled theory in M-theory, namely OM theory [9, 10]. The physical picture here is that the electric force due to the near-critical electric field C_{012} balances the open membrane tension such that we end up with a finite proper tension in the decoupling from the bulk (i.e. sending the eleven-dimensional Planck length to zero). Furthermore the compactification of OM theory on a magnetic circle yields a $(1+4)$-dimensional non-commutative Yang-Mills theory (NCYM) with a space-space noncommutativity. Recall that the $(1+4)$-dimensional NCYM is not renormalizable and new degrees of freedom enter when the energy reaches around $1/g^2_{\rm YM}$. Thus the OM theory provides the high energy completion of the NCYM. On the other hand, the compactification of OM theory on an electric circle gives a $(1+4)$-dimensional NCOS theory with a space-time noncommutativity. Thus OM theory provides a description of the NCOS theory when its coupling is strong. Therefore the $(1+4)$-dimensional NCYM and NCOS have a unified origin in six dimensions: OM theory. The relation of OM theory to the NCOS is quite similar to that between M-theory and IIA superstring theory.

In general, a constant C_{012}-field can be traded to a constant M5-brane worldvolume 3-form field strength[1] H_{012}. From the M5-brane worldvolume perspective, an M5-brane with a constant H_{012} represents a non-threshold bound state of this M5-brane with delocalized M2-branes along two of the 5 spatial directions of the M5-brane. The gravity configuration for this (M5, M2) bound state was given in [11]. Recently the gravity configuration of a more general non-threshold (M5, M2, M2′, MW) bound state has been constructed in [12] by uplifting the known (D4, D2, D2′, D0) bound state solution of the 10-dimensional type-IIA supergravity [13, 14] to 11-dimensions. This bound state preserves also 1/2 spacetime supersymmetries just as the (M5, M2) bound state does. Here MW stands for M-wave and M2 and M2' denote the corresponding delocalized M2-branes, respectively, along directions orthogonal to each other on the M5-brane worldvolume.

Given that the asymptotic configuration of the (M5, M2, M2′, MW) bound state is related to that of the (M5, M2) bound state by a finite Lorentz boost and the two bound states preserve the same number of spacetime supersymmetry, one must be wondering if the two bound states are in general related to each other by such a boost. This is not obvious at all if one simply examines the corresponding gravity configurations of the two bound states. We will show in the following section that this is indeed true. Given this result, it is natural for us to test whether the recently proposed boost-related V-duality, first for NCOS in [15] and then elaborated and extended to more general cases in [16], works for OM theory as well. We will investigate this in the present paper and the answer is positive.

This paper is organized as follows: in section 2, we show that the (M5, M2, M2′, MW) bound state is indeed related to the (M5, M2) bound state by a finite Lorentz boost along a direction perpendicular to the M2-brane. In section 3, we show that insisting a decoupled OM theory allows only an infinitesimal Lorentz boost which appears to be a galilean one for the decoupled OM theory. In other words, the al-

[1] The non-linear self-duality relation implies the presence of H_{345} as well. The same should be true for C_{345} (see also the discussion given in [8]).

lowed non-trivial spacetime boosts connecting a deformed OM theory to the original one are galilean ones;[2] conjecturally they result in physically equivalent theories. We will discuss the V-duality for OM theory both from the gravity perspective and from the M5-brane worldvolume perspective. In section 4, we consider the compactification of the deformed OM theory on either an "electric" circle or a "magnetic" circle.[3] We find that the V-duality for OM theory gives the corresponding known V-duality for either NCOS or NCYM discussed in [15, 16]. We conclude this paper in section 5.

2. The (M5, M2, M2′, MW) bound state

In this section we show that the (M5, M2, M2′, MW) bound state, recently constructed in [12] by uplifting the known (D4, D2, D2′, D0) bound state to 11-dimensions, can also be obtained from the (M5, M2) bound state by a finite Lorentz boost along a M5-brane direction perpendicular to the M2-brane. We start with the supergravity solution of the (M5, M2) bound state [11],

$$\begin{aligned} ds^2 &= H^{-1/3}h^{-1/3}\Big[-dt^2 + (dx^1)^2 + (dx^2)^2 + h\Big((dx^3)^2 + (dx^4)^2 + (dx^5)^2\Big) + \\ &\qquad + H\left(dr^2 + r^2 d\Omega_4^2\right)\Big] , \\ l_{\rm p}^3 C &= H^{-1}\sin\alpha dt\wedge dx^1\wedge dx^2 - H^{-1}h\tan\alpha dx^3\wedge dx^4\wedge dx^5 , \\ F_4 &= 3\pi N l_{\rm p}^3 \epsilon_4 , \end{aligned} \tag{2.1}$$

where $l_{\rm p}$ is the Planck constant in 11-dimensions, N is the number of M5-branes in the bound state, ϵ_4 is the volume form of 4-sphere with a unit radius, and the function h and the harmonic function H are defined as

$$H = 1 + \frac{R^3}{\cos\alpha\, r^3}, \qquad h^{-1} = H^{-1}\sin^2\alpha + \cos^2\alpha , \tag{2.2}$$

with $R^3 = \pi N l_{\rm p}^3$.

We now Lorentz boost this system along the x^5-direction with a boost parameter γ,

$$t \longrightarrow t\cosh\gamma - x^5\sinh\gamma , \qquad x^5 \longrightarrow x^5\cosh\gamma - t\sinh\gamma , \tag{2.3}$$

[2]There are some subtleties here when the boost is not perpendicular to the original M2 directions. We will discuss this in [17]. The "non-trivial spacetime boost" here means those which are not in the symmetry group $SO(1,2)$ of the original OM theory.

[3]Here the "electric" or "magnetic" circle is defined with respect to the original OM theory without the boost. In other words, an "electric" direction is either the x^1 or the x^2 direction while a "magnetic" direction is any of the x^3, x^4, x^5 directions. Here the definition for either "electric" or "magnetic" direction is not perfect since we have non-vanishing C_{034}. But these are just convenient names which will be used later when we consider compactifications of the deformed OM theory.

and we end up with

$$
\begin{aligned}
ds^2 &= H^{-1/3}h^{-1/3}\Big[-dt^2+(dx^1)^2+(dx2)^2+(dx^5)^2+h\left((dx^3)^2+(dx^4)^2\right)+\\
&\qquad\qquad + (h-1)\left(\cosh\gamma dx^5-\sinh\gamma dt\right)^2+H\left(dr^2+r^2d\Omega_4^2\right)\Big],\\
l_{\rm p}^3C &= H^{-1}\Big(\sin\alpha\cosh\gamma dt\wedge dx^1\wedge dx^2-\sin\alpha\sinh\gamma dx^1\wedge dx^2\wedge dx^5+\\
&\qquad + h\tan\alpha\sinh\gamma dt\wedge dx^3\wedge dx^4-\\
&\qquad - h\tan\alpha\cosh\gamma dx^3\wedge dx^4\wedge dx^5\Big),
\end{aligned}
\tag{2.4}
$$

where the 4-form field strength F_4 as given in (2.1) remains the same. Defining

$$
\begin{aligned}
\frac{\sin\theta_2}{\cos\theta_1} &= \sin\alpha\cosh\gamma, \qquad \tan\theta_1=\sin\alpha\sinh\gamma,\\
\frac{\sin\theta_1}{\cos\theta_2} &= \tan\alpha\sinh\gamma, \qquad \tan\theta_2=\tan\alpha\cosh\gamma,
\end{aligned}
\tag{2.5}
$$

we then have[4]

$$
\cos\alpha=\frac{\cos\theta_2}{\cos\theta_1}, \qquad \cosh\gamma=\frac{\sin\theta_2}{\cos\theta_1}\frac{1}{\sin\alpha}, \qquad \sinh\gamma=\frac{\sin\theta_1}{\cos\theta_2}\frac{1}{\tan\alpha}. \tag{2.6}
$$

Further if we set

$$
H'=1+\frac{R^3}{\cos\theta_1\cos\theta_2\, r^3}, \qquad h_i^{-1}=H'^{-1}\sin^2\theta_i+\cos^2\theta_i, \qquad i=1,2, \tag{2.7}
$$

we can derive

$$
H=H'h_1^{-1}, \qquad h^{-1}=h_1h_2^{-1}. \tag{2.8}
$$

Using the above relations, the solution (2.4) can be expressed as

$$
\begin{aligned}
ds^2 &= H'^{-1/3}h_1^{-1/3}h_2^{-1/3}\Big[-h_1dt^2+h_1\left((dx^1)^2+(dx^2)^2\right)+h_2\left((dx^3)^2+(dx^4)^2\right)+\\
&\qquad + (h_2-h_1)(\cosh\gamma dx^5-\sinh\gamma dt)^2+\\
&\qquad + H'\left(dr^2+r^2d\Omega_4^2\right)\Big].
\end{aligned}
\tag{2.9}
$$

Using the following three identities,

$$
\begin{aligned}
-h_1+(h_2-h_1)\sinh^2\gamma &= -1+h_1h_2\sin^2\theta_1\sin^2\theta_2\left(H'^{-1}-1\right)^2,\\
h_1+(h_2-h_1)\cosh^2\gamma &= h_1h_2,\\
-2(h_2-h_1)\cosh\gamma\sinh\gamma &= 2h_2h_1\left(H'^{-1}-1\right)\sin\theta_1\sin\theta_2,
\end{aligned}
\tag{2.10}
$$

[4] Here we obtain the boosted configuration only for $\cos\theta_2\le\cos\theta_1$. The case with $\cos\theta_2>\cos\theta_1$ can be obtained by the same boost but now on the (M5, M2) bound state with M2-branes along x^3x^4 directions.

we can re-express the boosted solution (2.4) as

$$ds^2 = H'^{-1/3} h_1^{-1/3} h_2^{-1/3} \Big[-dt^2 + h_1 \left((dx^1)^2 + (dx^2)^2\right) + h_2 \left((dx^3)^2 + (dx^4)^2\right) + \\ + h_1 h_2 \Big(dx^5 + \sin\theta_1 \sin\theta_2 (H'^{-1} - 1) dt\Big)^2 + \\ + H' \left(dr^2 + r^2 d\Omega_4^2\right) \Big] , \tag{2.11}$$

and

$$l_{\rm p}^3 C = H'^{-1} \Big(\frac{\sin\theta_2}{\cos\theta_1} h_1 dt \wedge dx^1 \wedge dx^2 + \frac{\sin\theta_1}{\cos\theta_2} h_2 dt \wedge dx^3 \wedge dx^4 - \\ - h_1 \tan\theta_1 dx^1 \wedge dx^2 \wedge dx^5 - h_2 \tan\theta_2 dx^3 \wedge dx^4 \wedge dx^5 \Big) , \\ F_4 = 3\pi N l_{\rm p}^3 \epsilon_4 . \tag{2.12}$$

Equations (2.11) and (2.12) together are nothing but the gravity configuration of the (M5, M2, M2′, MW) bound state that was recently constructed in [12] by uplifting the (D4, D2, D2′, D0) bound state solution [13, 14] to 11-dimensions.

3. The galilean nature of OM theory: V-duality

In this section we intend to study a possible OM theory decoupling limit for the boosted (M5,M2) system as given in (2.4). The usual OM theory decoupling limit as given in [9, 10] is actually with respect to a static (M5, M2) system with non-vanishing asymptotic C_{012} and C_{345}. The presence of C_{345} originates from a non-linear self-duality relation on the M5-brane worldvolume 3-form field strength [18, 19]. This decoupling limit requires a near-critical electric field C_{012} whose force almost balances the open membrane tension in a limit in which the bulk gravity decouples. As a result, the open membrane is confined within the M5-brane worldvolume with a finite tension which defines the scale for the OM theory.

For the boosted (M5,M2) system given in (2.4), one cannot blindly use the boosted electric component C_{012} to define the near-critical field limit, since in this case the net force acting on the open membrane is not merely due to this C_{012} and the other non-vanishing components of the background C-field such as C_{034} contribute to the net force as well. What we should account for is the total net force acting on the open membrane. Given the fact that we are considering a boosted system from the original static one, we must conclude that the near-critical field limit obtained for the static case remains the same even for the present boosted system.

The above indicates that a near-critical field limit is independent of a Lorentz boost. This in turn seemingly implies that a Lorentz boost is allowed for a decoupled OM theory. Our investigation in [17] tells that except for a static configuration, a near-critical field limit itself is in general not enough to define a decoupled theory.

In addition, either a well-defined decoupled gravity description or a proper open brane description (when available) is needed. In the following, we first discuss an OM theory decoupling limit for the boosted system in a gravity setup. We find that insisting a well-defined OM theory allows only infinitesimal Lorentz boost. This boost appears as a galilean one to the decoupled theory. This indicates that the V-duality discussed recently in [15, 16] holds for OM theory as well. We will check this using the proposed open membrane metric in [6, 10] as well. The conclusion remains the same.

As mentioned above, the near-critical field limit for the boosted system remains the same as that for (M5, M2). In other words, we have now ($\epsilon \to 0$)

$$l_{\rm p} = \epsilon l_{\rm eff}\,, \qquad \cos\alpha = \epsilon^{3/2}\,. \tag{3.1}$$

Note that the scalings for the asymptotic transverse coordinates should remain the same as for the case without boost. This gives $r = \epsilon^{3/2} u$ with fixed u. With these, insisting the decoupling of a flat bulk region from that of the M5-branes requires further the following:

$$x^{0,1,2} = \tilde{x}^{0,1,2}\,, \qquad x^{3,4,5} = \epsilon^{3/2}\tilde{x}^{3,4,5}\,, \qquad \gamma = \tilde{v}\epsilon^{3/2}\,, \tag{3.2}$$

where $\tilde{x}^\mu$ with $\mu = 0, 1, \cdots 5$ (here $x^0 = t$) and $\tilde{v}$ remain fixed. With the above, the gravity description of OM theory is now

$$\begin{aligned} ds^2 &= \epsilon^2 \left(\frac{u^3}{\pi N l_{\rm eff}^3}\right)^{1/3} \tilde{h}^{-1/3}\Big[- d\tilde{t}^2 + (d\tilde{x}^1)^2 + (d\tilde{x}^2)^2 + \tilde{h}((d\tilde{x}^3)^2 + \big(d\tilde{x}^4)^2\big) + \\ &\qquad\qquad + \tilde{h}(d\tilde{x}^5 - \tilde{v}d\tilde{t})^2 + \frac{\pi N l_{\rm eff}^3}{u^3}\left(du^2 + u^2 d\Omega_4^2\right)\Big]\,, \\ C &= \frac{u^3}{\pi N l_{\rm eff}^6} d\tilde{t}\wedge d\tilde{x}^1 \wedge d\tilde{x}^2 - \frac{u^3}{\pi N l_{\rm eff}^6}\tilde{h} d\tilde{x}^3 \wedge d\tilde{x}^4 \wedge \left(d\tilde{x}^5 - \tilde{v} d\tilde{t}\right)\,, \end{aligned} \tag{3.3}$$

where $\tilde{h}^{-1} = 1 + u^3/\pi N l_{\rm eff}^3$. When $\tilde{v} = 0$, the gravity dual reduces to one for the usual OM theory [20].

The above results confirm our expectation. A well-defined decoupled OM theory requires the boost $\gamma = \tilde{v}\epsilon^{3/2}$ to be infinitesimally small. This appears to be a galilean boost which relates the usual OM theory to the present one through

$$\tilde{t} \to \tilde{t}\,, \qquad \tilde{x}^5 \longrightarrow \tilde{x}^5 - \tilde{v}\tilde{t}\,. \tag{3.4}$$

The background C-field (or the M5-brane worldvolume 3-form field strength H) is needed to define the OM theory. The presence of this field breaks the M5-brane worldvolume Lorentz symmetry. But this breaking is spontaneous. We therefore expect that the OM theories resulting from the respective decouplings in such related backgrounds are physically equivalent. Along the same line as discussed in [16], we have therefore V-duality holding true for OM theory.

JHEP02(2001)024

For the rest of this section, we make an independent check of what has been achieved above using the effective open membrane metric[5] (the one seen by the open membrane) recently proposed in [6, 10]. The nonlinear self-duality condition for the M5-brane worldvolume 3-form field strength H reads [19]

$$\frac{\sqrt{-g}}{6}\epsilon_{\mu\nu\rho\sigma\lambda\tau}H^{\sigma\lambda\tau} = \frac{1+K}{2}(G^{-1})_{\mu}^{\ \lambda}H_{\nu\rho\lambda}\,, \tag{3.5}$$

where $g_{\mu\nu}$ is the induced metric on the M5-brane, $\epsilon^{012345} = 1$ and the scalar K and the tensor G are given by

$$\begin{aligned} K &= \sqrt{1+\frac{l_{\rm p}^6}{24}H^2}\,, \\ G_{\mu\nu} &= \frac{1+K}{2K}\left(g_{\mu\nu}+\frac{l_{\rm p}^6}{4}H^2_{\mu\nu}\right). \end{aligned} \tag{3.6}$$

It is understood that the indices in the above equations are raised or lowered using the induced metric. It was argued in [6, 10] that the symmetric tensor $G_{\mu\nu}$ is related to the effective open membrane metric up to a conformal factor. With an appropriate choice of the conformal factor, this proposed metric for OM theory was shown in [10] to reduce to the respective effective open string metric upon dimensional reductions.

For the boosted (M5,M2) given in (2.4), the asymptotic bulk metric and the C-field along the M5-brane directions are

$$\begin{aligned} ds^2 &= -dt^2 + g_1^2\left((d\tilde{x}^1)^2+(d\tilde{x}^2)^2\right) + g_2^2\left((d\tilde{x}^3)^2+(d\tilde{x}^4)^2\right) + g_3^2(d\tilde{x}^5)^2\,, \\ l_{\rm p}^3 C &= g_1^2\sin\alpha\cosh\gamma dt\wedge d\tilde{x}^1\wedge d\tilde{x}^2 - g_1^2 g_3\sin\alpha\sinh\gamma d\tilde{x}^1\wedge d\tilde{x}^2\wedge d\tilde{x}^5 - \\ &\quad - g_2^2 g_3\tan\alpha\cosh\gamma d\tilde{x}^3\wedge d\tilde{x}^4\wedge d\tilde{x}^5 + g_2^2\tan\alpha\sinh\gamma dt\wedge d\tilde{x}^3\wedge d\tilde{x}^4\,, \end{aligned} \tag{3.7}$$

where we have scaled the coordinates according to the symmetry of system under consideration.

Recall that the C-field can be traded for the H-field. With this in mind and from (3.7), we have

$$G'_{\mu\nu} \equiv g_{\mu\nu}+\frac{l_{\rm p}^6}{4}H^2_{\mu\nu} = \begin{pmatrix} \mathcal{A} & 0 & 0 & 0 & 0 & \mathcal{B} \\ 0 & \frac{g_1^2(3+\cos 2\alpha)}{4} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{g_1^2(3+\cos 2\alpha)}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{g_2^2(2+\tan^2\alpha)}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{g_2^2(2+\tan^2\alpha)}{2} & 0 \\ \mathcal{B} & 0 & 0 & 0 & 0 & \mathcal{C} \end{pmatrix} \tag{3.8}$$

and

$$l_{\rm p}^6 H^2 = 6\sin^2\alpha\tan^2\alpha\,, \tag{3.9}$$

[5]This metric was proposed to play a similar role as the effective open string metric given by Seiberg and Witten [1]. However, unlike the open string metric, this metric cannot be derived directly due to our inability at present to quantize the membrane.

JHEP02(2001)024

where

$$\mathcal{A} = \frac{-2 + \sin^2\alpha\cosh^2\gamma + \tan^2\alpha\sinh^2\gamma}{2}\,, \qquad \mathcal{B} = -\frac{g_3(3+\cos 2\alpha)\sinh 2\gamma\tan^2\alpha}{8}\,,$$
$$\mathcal{C} = \frac{g_3^2(2 + \sin^2\alpha\sinh^2\gamma + \cosh^2\gamma\tan^2\alpha)}{2}\,. \tag{3.10}$$

Following [10], we propose the tensor $G'_{\mu\nu}$ to be conformally related to the open membrane metric $\tilde{G}_{\mu\nu}$ as

$$\tilde{G}_{\mu\nu} = f\left(l_{\rm p}^6 H^2\right) G'_{\mu\nu}\,, \tag{3.11}$$

where f is a function of $l_{\rm p}^6 H^2$ and is introduced such that the open membrane metric is finite in units of l_p^2. However, the precise form for this function f has not been determined. Fortunately, this does not prevent us from making the present discussion.

Since the tensor $G'_{\mu\nu}$ differs from the true open membrane metric only by a conformal factor, we expect that at least its diagonal elements should scale the same way since they survive when the boost is set to zero. Further inspecting the tensor $G'_{\mu\nu}$, we find that $G'_{11}, G'_{22}, G'_{33}, G'_{44}$ are independent of the boost as they should be. With the near-critical field limit (3.1), we have now $g_1 \sim 1, g_2 \sim \epsilon^{3/2}$. These further imply that the $\mathcal{A}$ and $\mathcal{C}$ should be fixed, too. The former determines that the boost must be $\gamma = \tilde{v}\epsilon^{3/2}$ with fixed $\tilde{v}$, i.e. infinitesimally small. The latter determines $g_3 \sim \epsilon^{3/2}$. These in turn imply that $\mathcal{B}$ is also fixed. If we set from the above the following,

$$g_1 = 1\,, \qquad g_2 = g_3 = \epsilon^{3/2}\,, \qquad \gamma = \tilde{v}\epsilon^{3/2}\,, \tag{3.12}$$

we have

$$G'_{\mu\nu} = \frac{1}{2}\begin{pmatrix} -1+\tilde{v}^2 & 0 & 0 & 0 & 0 & -\tilde{v} \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ -\tilde{v} & 0 & 0 & 0 & 0 & 1 \end{pmatrix}. \tag{3.13}$$

The above $G'_{\mu\nu}$ says that we can set $f = 2\epsilon^2$ such that

$$l_{\rm p}^{-2}\tilde{G}_{\mu\nu} = 2 l_{\rm eff}^{-2} G'_{\mu\nu}\,. \tag{3.14}$$

Once again the constant $l_{\rm eff}$ sets the effective scale for the decoupled OM theory. The above open membrane metric $\tilde{G}_{\mu\nu}$ is related to the one without boost precisely by the galilean boost (3.4) as expected.

4. Compactifications of the deformed OM theory

In this section, we will show that the V-duality for NCOS and NCYM discussed in [15, 16] can be obtained from the V-duality for OM theory discussed in the previous section through compactifications and T-dualities. As mentioned in [16], the V-

JHEP02(2001)024

duality survives S-duality there. Since U-dualities for the big M-theory are believed to be inherited to the little m-theory without gravity, our study so far indicates that the U-duality related decoupled theory has V-duality if the original decoupled theory does. This further enforces our conclusion made in [16] and to be reported in a forthcoming paper [17] that V-duality holds in general in the little m-theory.

4.1 Compactification on an electric circle

In this subsection, we consider first the compactification of the deformed OM theory on an electric circle (on the direction $\tilde{x}^2$ for definiteness) following [9]. This will give a family of (1+4)-dimensional NCOS theories related by V-duality. Further applying T-duality on, for definiteness, $\tilde{x}^3$, we end up with a family of $(1+3)$-dimensional NCOS related by V-duality which was discussed specifically in [15, 16]. In other words, the V-duality for OM theory is inherited to its descendant theories as well.

Consider M-theory on $R^{10} \times S^1$(the circle is in the x^2 direction) with M5-branes wrapping the circle. Scale all bulk moduli as in the previous section for OM theory,[6] i.e.

$$\begin{aligned}
l_{\rm p} &= \epsilon^{1/3} l_{\rm eff}\,, \qquad \cos\alpha = \epsilon^{1/2}\,, \qquad \gamma = \tilde{v}\epsilon^{1/2}\,,\\
g_{\mu\nu} &= \eta_{\mu\nu}(\mu,\nu = 0,1,2)\,, \qquad g_{ij} = \epsilon\delta_{ij}(i,j = 3,4,5)\,,\\
g_{mn} &= \epsilon\delta_{mn}(m,n = \text{transverse})\,,\\
l_{\rm p}^3 C &= \sin\alpha\cosh\gamma d\tilde{t}\wedge d\tilde{x}^1\wedge d\tilde{x}^2 - \epsilon^{1/2}\sin\alpha\sinh\gamma d\tilde{x}^1\wedge d\tilde{x}^2\wedge d\tilde{x}^5 -\\
&\quad - \epsilon^{3/2}\tan\alpha\cosh\gamma d\tilde{x}^3\wedge d\tilde{x}^4\wedge d\tilde{x}^5 + \epsilon\tan\alpha\sinh\gamma d\tilde{t}\wedge d\tilde{x}^3\wedge d\tilde{x}^4\,, \qquad (4.1)
\end{aligned}$$

where we have kept the cosines and sines as well as cosh's and sinh's in C for later convenience.

The relation between M-theory and IIA implies that the above decoupled system is equivalent to a decoupled one of D4-branes in IIA theory with

$$\begin{aligned}
\alpha' &= \epsilon\alpha'_{\rm eff}\,, \qquad g_s = \frac{G_o^2}{\sqrt{\epsilon}}\,, \qquad \cos\alpha = \epsilon^{1/2}\,, \qquad \gamma = \tilde{v}\epsilon^{1/2}\,,\\
g_{\mu\nu} &= \eta_{\mu\nu}\ (\mu,\nu = 0,1)\,, \qquad g_{ij} = \epsilon\delta_{ij}\ (i,j = 3,4,5)\,,\\
g_{mn} &= \epsilon\delta_{mn}\ (m,n = \text{transverse})\,,\\
2\pi\alpha' B &= \sin\alpha\cosh\gamma d\tilde{t}\wedge d\tilde{x}^1 + \epsilon^{1/2}\sin\alpha\sinh\gamma d\tilde{x}^1\wedge d\tilde{x}^5\,, \qquad (4.2)
\end{aligned}$$

where

$$\alpha'_{\rm eff} = \frac{l_{\rm eff}^3}{R_2}\,, \qquad G_o^2 = \left(\frac{R_2}{l_{\rm eff}}\right)^{3/2}\,, \qquad (4.3)$$

with R_2 the proper (also the coordinate) radius of the circle.

[6] For convenience of the present discussion, we have replaced the scaling parameter ϵ used in the previous section by $\epsilon^{1/3}$.

The above is nothing but the scaling limit for $(1+4)$-dimensional NCOS. To see the V-duality, let us calculate the open string moduli using Seiberg-Witten relation [1] and we have for the open string metric

$$G_{\alpha\beta} = \epsilon \begin{pmatrix} -(1-\tilde{v}^2) & 0 & 0 & 0 & -\tilde{v} \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -\tilde{v} & 0 & 0 & 0 & 1 \end{pmatrix}, \tag{4.4}$$

and for the non-vanishing non-commutative parameters

$$\Theta^{01} = 2\pi\alpha'_{\mathrm{eff}}\,, \qquad \Theta^{15} = -2\pi\alpha'_{\mathrm{eff}}\tilde{v}\,. \tag{4.5}$$

In the above, $\alpha, \beta = 0, 1, 3, 4, 5$.

The above describes a family of $(1+4)$-dimensional NCOS theories which are related to the one with $\tilde{v} = 0$ by the following galilean boost:

$$\tilde{t} \to \tilde{t}\,, \qquad \tilde{x}^5 \to \tilde{x}^5 - \tilde{v}\tilde{t}\,. \tag{4.6}$$

In other words, we have the V-duality for $(1+4)$-dimensional NCOS as discussed in [15, 16] which is now obtained from that for OM theory.

T-duality of the above along, say, the $\tilde{x}^3$ direction with the coordinate radius R_3, will give a family of $(1+3)$-dimensional NCOS theories which are related to the one with $\tilde{v} = 0$ again by V-duality. The V-duality for this case was particularly studied in [15, 16]. Now we see that this V-duality can also be derived from that for OM theory. This T-duality leaves the non-commutative parameters intact and reduces trivially the open string metric in (4.4) to rank 4 through dropping $G_{3\beta}$ and $G_{\alpha 3}$. The changes are on the open string coupling and the T-dual radius (denoted as $\bar{G}_o$ and $\bar{R}_3$, respectively) as

$$\bar{R}_3 = \frac{\alpha'_{\mathrm{eff}}}{R_3}\,, \qquad \bar{G}_o^2 = \frac{G_o^2\sqrt{\alpha'_{\mathrm{eff}}}}{R_3} = \frac{R_2}{R_3}\,, \tag{4.7}$$

where we have used eq. (4.3) for the last equality in the second equation. We will not address potential subtleties(for examples, see [21]–[24]) related to the T-dualities of decoupled theories here.

4.2 Compactification on a magnetic circle

The compactification of the deformed OM theory on a magnetic circle[7] (say, a circle along the x^3 direction) follows the usual story to give a $(1+4)$-dimensional NCYM with a rank-2 non-commutative matrix. A further T-duality, say, along the x^2 direc-

[7]The compactification along x^5-direction seems to give the usual $(1+4)$-dimensional NCYM. The boost is now an internal momentum and appears to be invisible to the NCYM. For this reason, we consider here only the compactification along either the x^3 or x^4 direction.

tion gives $(1+3)$-dimensional NCYM with again a rank-2 non-commutative matrix. The V-duality for either $(1+4)$-dimensional or $(1+3)$-dimensional NCYM as discussed in [16], as we will see, follows from that for OM theory.

To begin with, consider M-theory again on $R^{10} \times S^1$ (but now the circle is along the x^3 direction) with M5-branes wrapping on this circle. The scaling limits for the bulk moduli for OM theory is again given by eq. (4.1). The relation between M-theory and IIA now implies that the above OM theory is equivalent to a decoupled theory of IIA theory as

$$\begin{aligned}
\alpha' &= \epsilon' \alpha'_{\text{eff}}\,, \qquad g_s = \epsilon'^{1/2}\left(\frac{R_3}{l_{\text{eff}}}\right)^{3/2}, \qquad \cos\alpha = \epsilon'\,, \qquad \gamma = \tilde{v}\epsilon'\,, \\
g_{\mu\nu} &= \eta_{\mu\nu}\ (\mu,\nu = 0,1,2)\,, \qquad g_{ij} = \epsilon'^2 \delta_{ij}\ (i,j = 4,5)\,, \\
g_{mn} &= \epsilon'^2 \delta_{mn}\ (m,n = \text{transverse})\,, \\
2\pi\alpha' B &= -\epsilon' \tan\alpha \sinh\gamma\, d\tilde{t} \wedge d\tilde{x}^4,\ -\epsilon'^2 \tan\alpha \cosh\gamma\, d\tilde{x}^4 \wedge d\tilde{x}^5\,,
\end{aligned} \tag{4.8}$$

where we have set the new scaling parameter $\epsilon' = \epsilon^{1/2}$ and

$$\alpha'_{\text{eff}} = \frac{l^3_{\text{eff}}}{R_3}\,. \tag{4.9}$$

The above scaling limit is the one just for (1+4)-dimensional NCYM as expected. We actually have a family of $(1+4)$-dimensional NCYM theories which are related to the one corresponding to $\tilde{v} = 0$ by a galilean boost given in (4.6). We can see this easily by computing the Seiberg-Witten open string moduli, for the metric,

$$G_{\alpha\beta} = \begin{pmatrix} -(1-\tilde{v}^2) & 0 & 0 & 0 & -\tilde{v} \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -\tilde{v} & 0 & 0 & 0 & 1 \end{pmatrix}, \tag{4.10}$$

and for the non-vanishing non-commutative parameter

$$\Theta^{45} = 2\pi\alpha'_{\text{eff}}\,. \tag{4.11}$$

In the above, $\alpha, \beta = 0, 1, 2, 4, 5$.

The above open string metric is related to the one with $\tilde{v} = 0$ by the galilean boost (4.6) and the non-commutative parameter remains intact under this boost as it should be. We therefore shows that the V-duality for NCYM as discussed in [16] can also be obtained from that for OM theory. A T-duality of the above along, say, the $\tilde{x}^2$-direction will give a family of $(1+3)$-dimensional NCYM theories which are again related to the one with $\tilde{v} = 0$ by the V-duality discussed in [16]. But we have it here from that for OM theory. This T-duality does not touch the non-commutative parameter but reduces trivially the open string metric to a rank-4 one through dropping $G_{2\beta}$ and $G_{\alpha 2}$. The changes are on the closed string coupling, the gauge coupling and the radius of circle in the T-dual, i.e. the x^2-direction. If we denote the T-dual closed string coupling, coordinate radius and the gauge coupling as $\bar{g}_s$, $\bar{R}_2$ and $\bar{g}^2_{\rm YM}$, respectively (the original ones as $g_s, R_2, g^2_{\rm YM}$), we have first

$$\bar{R}_2 = \frac{\alpha'_{\rm eff}}{R_2}\,, \qquad \bar{g}_s = g_s \epsilon'^{1/2}\frac{\sqrt{\alpha'_{\rm eff}}}{R_2} = \epsilon' \frac{R_3}{R_2}\,, \tag{4.12}$$

where we have used g_s and $\alpha'_{\rm eff}$ given in eqs. (4.8) and (4.9) in the last equality of the second equation above. The Yang-Mills coupling for the $(1+4)$-dimensional NCYM can be calculated as

$$g^2_{\rm YM} = (2\pi)^2 g_s \sqrt{\alpha'}\left(\frac{\det G}{\det(g + 2\pi\alpha' B)}\right)^{1/2} = 4\pi^2 R_3\,. \tag{4.13}$$

The Yang-Mills coupling for the $(1+3)$-dimensional NCYM is related to the above as

$$\bar{g}^2_{\rm YM} = \frac{\bar{g}_s}{2\pi g_s\sqrt{\alpha'}} g^2_{\rm YM} = 2\pi\frac{R_3}{R_2}\,, \tag{4.14}$$

where we have used the relation between $\bar{g}_s$ and g_s as given above.

5. Conclusion

We show in this paper that the (M5, M2, M2′, MW) bound state of M-theory constructed recently in [12] through uplifting the (D4, D2, D2′, D0) bound state of IIA theory to eleven dimensions is actually related to the (M5, M2) bound state of M-theory by a finite Lorentz boost along a M5-brane direction perpendicular to the M2-brane. This motivates us to consider the V-duality for OM theory. We find indeed that the V-duality holds for OM theory. The meaning of this is that the allowed non-trivial[8] spacetime boost on a given OM theory is a galilean one, and such related OM theories are also physically equivalent. Everything here goes along the same line as what has been discussed in [15, 16] for NCOS and NCYM.

We further show that upon compactification on either an electric or a magnetic circle (as well as T-dualities), the V-duality for OM theory gives that for either NCOS or NCYM. This enforces our conclusion made in [16] and to be reported in detail in a forthcoming paper [17] that V-duality holds in general for little m-theory without gravity. It is also interesting to seek connections between V-duality discussed here and the recent work on the non-relativistic closed string theory (NRCS) and its strong coupling dual, i.e. the galilean membrane theory[9] [23, 24, 25]. We wish to report this in [17].

[8]By "non trivial", we mean that the boost does not belong to the symmetry group of the original decoupled OM theory. For example, we do not consider the boost along x^1 or x^2 directions.

[9]By definition, the NCRS and its strong coupling dual require the directions of the respective background field to be compactified but not the presence of the corresponding background brane.

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References

[1] N. Seiberg and E. Witten, *String theory and noncommutative geometry*, *J. High Energy Phys.* **09** (1999) 032 [hep-th/9908142].

[2] N. Seiberg, L. Susskind and N. Toumbas, *Strings in background electric field, space/time noncommutativity and a new noncritical string theory*, *J. High Energy Phys.* **06** (2000) 021 [hep-th/0005040].

[3] R. Gopakumar, J. Maldacena, S. Minwalla and A. Strominger, *S-duality and noncommutative gauge theory*, *J. High Energy Phys.* **06** (2000) 036 [hep-th/0005048].

[4] C.-S. Chu, P.-M. Ho and M. Li, *Matrix theory in a constant c field background*, *Nucl. Phys.* **B 574** (2000) 275 [hep-th/9911153].

[5] S. Chakravarty, K. Dasgupta, O.J. Ganor and G. Rajesh, *Pinned branes and new non Lorentz invariant theories*, *Nucl. Phys.* **B 587** (2000) 228 [hep-th/0002175].

[6] E. Bergshoeff, D.S. Berman, J.P. van der Schaar and P. Sundell, *A noncommutative M-theory five-brane*, *Nucl. Phys.* **B 590** (2000) 173 [hep-th/0005026].

[7] S. Kawamoto and N. Sasakura, *Open membranes in a constant c-field background and noncommutative boundary strings*, *J. High Energy Phys.* **07** (2000) 014 [hep-th/0005123].

[8] D.S. Berman and P. Sundell, *Flowing to a noncommutative (OM) five brane via its supergravity dual*, *J. High Energy Phys.* **10** (2000) 014 [hep-th/0007052].

[9] R. Gopakumar, S. Minwalla, N. Seiberg and A. Strominger, *OM theory in diverse dimensions*, *J. High Energy Phys.* **08** (2000) 008 [hep-th/0006062].

[10] E. Bergshoeff, D.S. Berman, J.P. van der Schaar and P. Sundell, *Critical fields on the M5-brane and noncommutative open strings*, *Phys. Lett.* **B 492** (2000) 193 [hep-th/0006112].

[11] J.M. Izquierdo, N.D. Lambert, G. Papadopoulos and P.K. Townsend, *Dyonic membranes*, *Nucl. Phys.* **B 460** (1996) 560 [hep-th/9508177].

[12] E. Bergshoeff, R.-G. Cai, N. Ohta and P.K. Townsend, *M-brane interpolations and (2,0) renormalization group flow*, *Phys. Lett.* **B 495** (2000) 201 [hep-th/0009147].

[13] J.C. Breckenridge, G. Michaud and R.C. Myers, *More D-brane bound states*, *Phys. Rev.* **D 55** (1997) 6438 [hep-th/9611174].

[14] T. Harmark and N.A. Obers, *Phase structure of non-commutative field theories and spinning brane bound states*, *J. High Energy Phys.* **03** (2000) 024 [hep-th/9911169].

[15] G.-H. Chen and Y.-S. Wu, *Comments on noncommutative open string theory: v-duality and holography*, hep-th/0006013.

[16] R.-G. Cai, J.-X. Lu and Y.-S. Wu, *The galilean nature of V-duality for noncommutative open string and Yang-Mills theories*, hep-th/0012239.

[17] R.G. Cai, J.X. Lu and Y.S. Wu, to appear.

[18] M. Perry and J.H. Schwarz, *Interacting chiral gauge fields in six dimensions and Born-Infeld theory*, *Nucl. Phys.* **B 489** (1997) 47 [hep-th/9611065].

[19] P.S. Howe, E. Sezgin and P.C. West, *Covariant field equations of the M-theory five-brane*, *Phys. Lett.* **B 399** (1997) 49 [hep-th/9702008]; *The six-dimensional self-dual tensor*, *Phys. Lett.* **B 400** (1997) 255 [hep-th/9702111].

[20] T. Harmark, *Open branes in space-time non-commutative little string theory*, *Nucl. Phys.* **B 593** (2001) 76 [hep-th/0007147].

[21] I.R. Klebanov and J. Maldacena, $1+1$ *dimensional ncos and its* $\mathrm{U}(N)$ *gauge theory dual*, hep-th/0006085.

[22] T. Kawano and S. Terashima, *S-duality from OM-theory*, *Phys. Lett.* **B 495** (2000) 207 [hep-th/0006225].

[23] U.H. Danielsson, A. Guijosa and M. Kruczenski, *Newtonian gravitons and D-brane collective coordinates in wound string theory*, hep-th/0012183.

[24] U.H. Danielsson, A. Guijosa and M. Kruczenski, *IIA/B, wound and wrapped*, *J. High Energy Phys.* **10** (2000) 020 [hep-th/0009182].

[25] J. Gomis and H. Ooguri, *Non-relativistic closed string theory*, hep-th/0009181.

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